Quantum Mechanics as Classical Physics

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1 A Series of Solutions to the Measurement Problem

According to *Everettian quantum mechanics*, in one of its simpler forms, the wave function is all there is and the evolution of the wave function is always determined by the Schrödinger equation,

$$i\hbar\frac{\partial}{\partial t}\Psi(\vec{x}_1,\vec{x}_2,...,t) = \left(\sum_k \frac{-\hbar^2}{2m_k}\nabla_k^2 + V(\vec{x}_1,\vec{x}_2,...,t)\right)\Psi(\vec{x}_1,\vec{x}_2,...,t)\ . \tag{1.1}$$

Bohmian mechanics adds something to the story. In addition to the unitarily evolving wave function, there are particles with definite positions. The wave function pushes particles around by a specified law,

$$\vec{v}_k(t) = \frac{\hbar}{m_k}\text{Im}\left[\frac{\vec{\nabla}_k\Psi(\vec{x}_1,\vec{x}_2,...,t)}{\Psi(\vec{x}_1,\vec{x}_2,...,t)}\right]\ . \tag{1.2}$$

Prodigal QM adds more particles by adding more worlds to the Bohmian story. As before, the wave function always obeys the Schrödinger equation. In addition to the wave function, there are a large number of different worlds, although a finite number, each represented by a point in configuration space. Each world is guided by the single universal wave function in accordance with the Bohmian guidance equation and thus each world follows a Bohmian trajectory through configuration space. The collection of worlds can be described by a density function on configuration space, $\rho(\vec{x}_1,\vec{x}_2,...,t)$, normalized so that $\int d^3x_1 d^3x_2...\rho = 1$. By hypothesis, worlds are initially distributed so that

$$\rho = |\Psi|^2\ . \tag{1.3}$$

The velocity of the kth particle in a world with particle configuration $(\vec{x}_1,\vec{x}_2,...)$ at t is

$$\vec{v}_k(\vec{x}_1,\vec{x}_2,...,t) = \frac{\hbar}{m_k}\text{Im}\left[\frac{\vec{\nabla}_k\Psi(\vec{x}_1,\vec{x}_2,...,t)}{\Psi(\vec{x}_1,\vec{x}_2,...,t)}\right]\ . \tag{1.4}$$

From (1.1), (1.3), and (1.4) one can derive the following expression for the acceleration of the j-th particle which makes no reference to the wave function:

$$m_j\vec{a}_j = -\vec{\nabla}_j\left[\sum_k \frac{-\hbar^2}{2m_k}\left(\frac{\nabla_k^2\sqrt{\rho}}{\sqrt{\rho}}\right) + V\right]\ . \tag{1.5}$$

2 Newtonian QM

Reality consists of a large but finite number of worlds whose distribution in configuration space can be summed up by a density function $\rho(\vec{x}_1,\vec{x}_2,...,t)$. The velocities of the various particles in the various worlds are described by $\vec{v}_k(\vec{x}_1,\vec{x}_2,...,t)$. There is a single dynamical law, (1.5). In response to the particle motions, the distribution ρ shifts in accordance with a continuity equation,

$$\frac{\partial\rho}{\partial t} = -\sum_k \vec{\nabla}_k\cdot(\rho\vec{v}_k)\ . \tag{2.1}$$

In this theory, the wave function is not fundamental. But, one can be introduced to describe ρ and $\vec{v}_k$ by the relations specified in (1.3) and (1.4).

Everettian QM	
Ontology:	**Law**:
♦ universal wave function $\Psi(\vec{x}_1, \vec{x}_2, ..., t)$	♦ Schrödinger equation (1.1)

Bohmian QM	
Ontology:	**Laws**:
♦ universal wave function $\Psi(\vec{x}_1, \vec{x}_2, ..., t)$	♦ Schrödinger equation (1.1)
♦ particles with positions $\vec{x}_k(t)$ and velocities $\vec{v}_k(t)$	♦ guidance equation (1.2)

Prodigal QM	
Ontology:	**Laws**:
♦ universal wave function $\Psi(\vec{x}_1, \vec{x}_2, ..., t)$	♦ Schrödinger equation (1.1)
♦ particles in many worlds described by a world density $\rho(\vec{x}_1, \vec{x}_2, ..., t)$ and velocity fields $\vec{v}_k(\vec{x}_1, \vec{x}_2, ..., t)$	♦ guidance equation (1.4)

Newtonian QM	
Ontology:	**Law**:
♦ particles in many worlds described by a world density $\rho(\vec{x}_1, \vec{x}_2, ..., t)$ and velocity fields $\vec{v}_k(\vec{x}_1, \vec{x}_2, ..., t)$	♦ Newtonian force law (1.5)

3 Some Virtues: Probability and Time Reversal

In Newtonian QM the wave function is not fundamental. Instead, it is merely a way of describing the worlds which is related to ρ and $\vec{v}_k$ by (1.3) and (1.4). From (1.3) it follows that the number of worlds in a particular region of configuration space is always proportional to $|\Psi|^2$. At any time, most agents are in high amplitude regions. So, in typical measurement scenarios, most agents will see long-run frequencies which agree with the predictions of the Born Rule. Since one is generally not sure which world they are in, they should expect to be in a world in which Born Rule statistics are observed.

Newtonian QM explains why in quantum mechanics the wave function is complex conjugated under time reversal, $\Psi(\vec{x}_1, \vec{x}_2, ..., t) \longrightarrow \Psi^*(\vec{x}_1, \vec{x}_2, ..., -t)$. The time reversal operation acting on the particle trajectories takes the history $\rho(\vec{x}_1, \vec{x}_2, ..., t)$ and $\vec{v}_k(\vec{x}_1, \vec{x}_2, ..., t)$ to $\rho(\vec{x}_1, \vec{x}_2, ..., -t)$ and $-\vec{v}_k(\vec{x}_1, \vec{x}_2, ..., -t)$. Since the time-reversal operation flips all of the velocities, (1.4) ensures that it flips the complex phase of the wave function.

4 A Vice: Non-quantum States

In formulating Newtonian QM, I have treated the collection of worlds as a continuum, described by a density function and a collection of velocity fields. If there are only a handful of worlds, use of this continuum limit is not justified.

For any wave function $\Psi(t)$ obeying the Schrödinger equation (1.1), there exists a world-density $\rho(t)$ and a collection of velocity fields $\vec{v}_k(t)$ obeying (1.5) such that the relations between Ψ, ρ, and $\vec{v}_k$ expressed in (1.3) and (1.4) are satisfied at all times. The converse does not hold. There are some combinations of ρ and $\vec{v}_k$, that is, some ways the universe might be according to Newtonian QM, that do not correspond to any wave function. For one to be able to introduce a wave function which satisfies (1.3) and (1.4), the velocity fields must satisfy a quantization condition.

Quantization Condition Integrating the momenta of the particles along any closed loop in configuration space gives a multiple of Planck's constant.

$$\oint \left\{ \sum_k \left[m_k \vec{v}_k \cdot d\vec{\ell}_k \right] \right\} = nh \ . \tag{4.1}$$